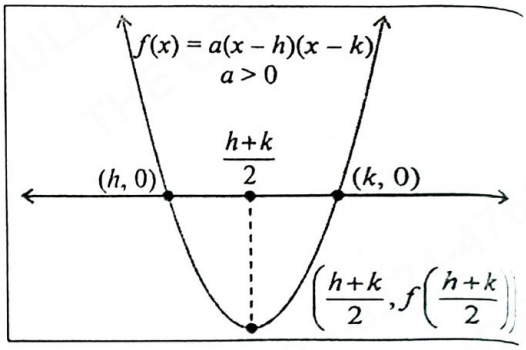
Exercise 1.1 IMPORTANT TOPICS & EXAMPLES

1.1 Graph of Quadratic Function Using Factors:

Method to Sketch the Graph

Suppose, after factorizing $f(x)=ax^2+bx+c$, we obtain $y=a(x-h)(x-k)$, where $h \le k$. To sketch the graph, follow the given steps:

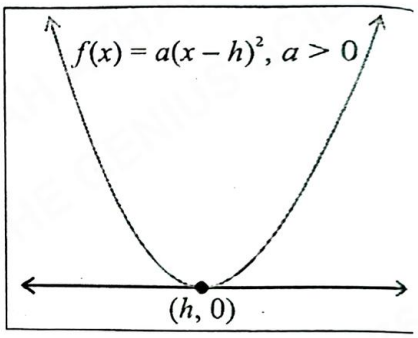


Case-I: if $h<k$

Step-1: The graph will cut the x-axis at if $x=h$ and $x=k$

Step-2: Find the value of y at $\frac{h+k}{2}$

The point $\left(\frac{h+k}{2}, f\left(\frac{h+k}{2}\right)\right)$ represents the vertex of the parabola.



Step-3: Determine the behavior of y as $x \to \pm\infty$.

- if $a>0$, then $y \to +\infty$.
- if $a<0$, then $y \to -\infty$.

This statement means the graph behaves the **same way on both ends** (left and right).

Step-4: Find the y-intercept:

Put $x=0$ in the function $y=a(x-h)(x-k)$, the graph will cut the y-axis at $y=ahk$

Case-II: if $h=k$, then the function becomes $y=a(x-h)^2$. To sketch the graph, follow these steps:

Step-1: The graph **touches the x-axis** at $x=h$, therefore $(h,0)$ is the vertex

Step-2: Then follow step 3 and 4 of Case-I to complete the sketch of the graph.

Important

$P(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-2}x^2+a_1x+a_n$ Is a polynomial of degree n, when n is a non-negative integer and $a_0 \neq 0$.

Example 1: *Sketch the graph of* $y=2(x-1)(x-3)$.

Sol: *To find the x-intercepts, set each factor equal to zero*

$x-1=0$ and $x-3=0$

$x=1$ and $x=3$

Hence, the graph meets the x-axis at the points $(1,0)$ and $(3,0)$.

To find y component of the vertex,

Put $x=\frac{1+3}{2}=2$ into the equation:

$y=2(2-1)(2-3)=-2$

So, the vertex of the graph is $(2,-2)$.

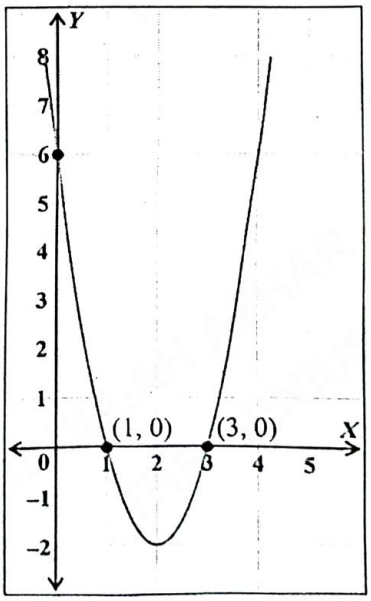
Since $a=2>$, so $y=\to+\infty$ as $x \to -\infty$ or $x \to +\infty$.

The graph cuts the y-axis at $x=0$ and

$y=2(-1)(-1)=6$

So, the y-intercept is $(0,6)$.

Using the intercepts and the vertex, we sketch the graph of the parabola as shown in figure.



Example 2: Sketch the graph of $y=-4x^2-8x-4$.

Sol: First, we factorize $y=-4x^2-8x-4$.

$$y=-4\left(x^2+2x+1\right).$$

$$y=-4(x+1)^2$$

To find the x-intercepts, set $x+1$ equal to zero:

$$x+1=0\Rightarrow x=-1$$

Hence, the graph meets the x-axis at the point $(-1,0)$.

Since the function is of the form $a(x-h)^2$, that is $y=-4(x+1)^2$, so $(-1,0)$ is its vertex.

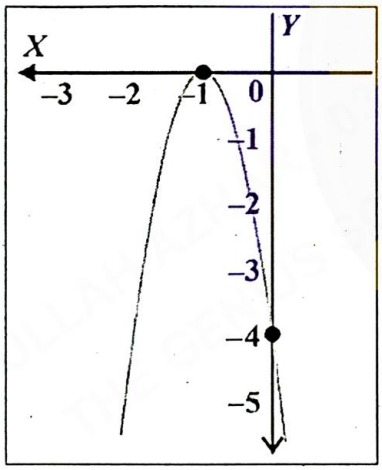
Since, $a=-4<0$, so $y\to-\infty$ or $x\to+\infty$

The graph cut the y-axis at $x=0$:

$$y=-4(0+1)^2=-4$$

So, the y-intercept is $(0,-4)$.

The sketch of the graph is shown in the fig.



1.2 Predicting a Quadratic Function from its Graph

A quadratic function $f(x)=ax^2+bx+c$ can be determined from its graph by using the **factor form** if the x-intercepts and one additional point on the curve are known.

If the graph crosses the x-axis at $x=h$ and $x=k$ then the function can be written as: $f(x)=a(x-h)(x-k)$

Here, a is a constant that determines the shape and direction of the parabola. To find a, substitute the coordinates of a third known point into the equation.

Example 3: Find the function whose graph is shown in Figure 1.5.

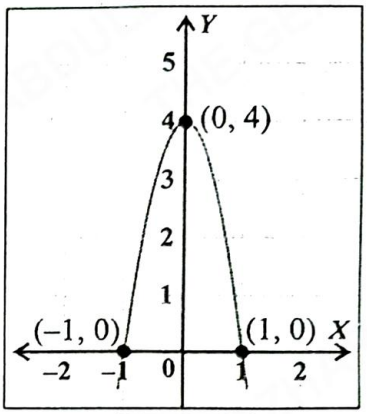


Figure 1.5

Sol: From figure it is clear that x-intercepts of the graph are $h=-1$ and $k=1$.

Therefore, the function can be written as:

$$y=a(x-(-1))(x-1)=a(x+1)(x-1) \text{ ---- (1)}$$

To find the values of a, substitute the point $(0,4)$ into equation(1):

$$4=a(0+1)(0-1)$$

$$4=-a\Rightarrow a=-4$$

Subtitling the values of a into the equation (1):

$$y=-4(x+1)(x-1)$$

$$y=-4\left(x^2-1\right)$$

$$y=-4x^2+4$$

Example 4: Find an equation of the graph of a parabola whose x-intercepts are $x=1$ and $x=2$ passes through the point $(3,4)$.

Sol: Since the x-intercepts are $x=1$ and $x=2$, the function can be written in factorized form as:

$$y=a(x-1)(x-2) \quad \text{---------(1)}$$

To find a, use the point $(3,4)$:

$$4=a(3-1)(3-2)=2a\Rightarrow a=\frac{4}{2}=2$$

Substituting the value of a, into the equation (1):

$$y=2(x-1)(x-2)$$

$$y=2\left(x^2-3x+2\right)=2x^2-6x+4$$

Exercise 1.1 SOLUTION

1. Sketch the graph of the following functions using the method of factorization

i) $y=3(x+1)(x-1)$

Sol: $y=3(x+1)(x-1)$ ..... (1)

x-intercepts

Step 1:

Put $y=0$ in (1)

$3(x+1)(x-1)=0$

$(x+1)(x-1)=0$, $3\neq 0$

$x+1=0 \Rightarrow \boxed{x=-1}$

$x-1=0 \Rightarrow \boxed{x=1}$

The graph cuts x-axis at $x=-1$, $x=1$

Step 2:

Put $x=\frac{-1+1}{2}=\frac{0}{2}=0$ in (1)

$y=3(0+1)(0-1) \Rightarrow y=-3$

So we have vertex $(0,-3)$

Step 3:

Since $a=3>0$

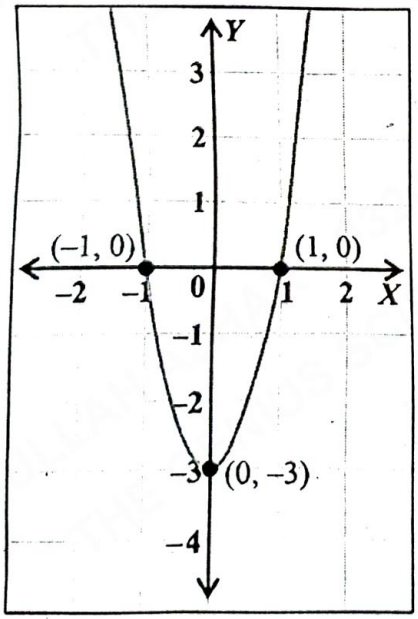
So, $y\to+\infty$ as $x\to\pm\infty$

Step 4:

Y-intercept

Put $x=0$ in (1)

The graph cuts y-axis at $y=-3$



ii) $f(x)=-2(x-1)(x-2)$

Sol. $y=-2(x-1)(x-2)$ ..... (1)

Step 1:

x-intercepts

put $y=0$ in (1)

$-2(x-1)(x-2)=0$

$(x-1)(x-2)=0$, $2\neq 0$

$x-1=0 \Rightarrow \boxed{x=1}$

$x-2=0 \Rightarrow \boxed{x=2}$

The graph cuts x-axis at $x=1$ and $x=2$

Step 2:

Put $x=\frac{1+2}{2}=\frac{3}{2}$ in (1)

$y=-2\left(\frac{3}{2}-1\right)\left(\frac{3}{2}-2\right) \Rightarrow y=-2\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)=\frac{1}{2}$

So, we have vertex $\left(\frac{3}{2},\frac{1}{2}\right)$

Step 3:

Since $a=-2<0$

So, $y\to-\infty$ as $x\to\pm\infty$

Step 4:

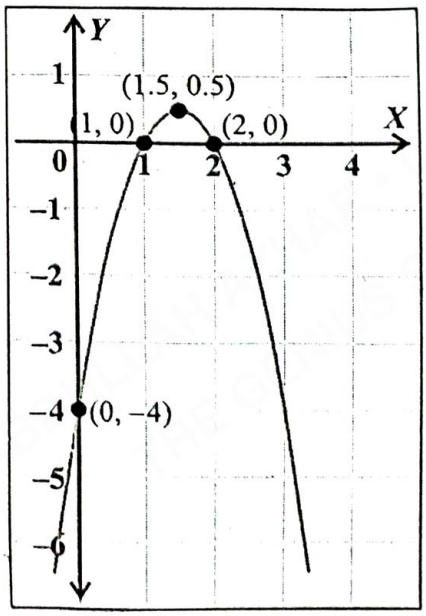
y-intercept

put $x=0$ in (1)

$y=-2(0-1)(0-2)$

$y=-2(-1)(-2) \Rightarrow y=-4$

The graph cuts y-axis at $y=-4$



iii) $y=(2x-1)(2x+1)$

Sol: $y=(2x-1)(2x+1)$

$$y=2\left(x-\frac{1}{2}\right)2\left(x+\frac{1}{2}\right)$$

$$y=4\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right) \quad \text{.....(1)}$$

Step 1: x-intercepts
Put $y=0$ in (1)

$$4\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)=0$$

$$\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)=0 \ , \ 4\neq 0$$

$$x-\frac{1}{2}=0 \Rightarrow \boxed{x=\frac{1}{2}}$$

$$x+\frac{1}{2}=0 \Rightarrow \boxed{x=\frac{-1}{2}}$$

The graph cuts x-axis at $x=\frac{1}{2}$ and $x=\frac{-1}{2}$

Step 2:

Put $x=\frac{\frac{1}{2}+\left(\frac{-1}{2}\right)}{2}=\frac{0}{2}=0$ in (1)

$$y=4\left(0-\frac{1}{2}\right)\left(0+\frac{1}{2}\right)=-\frac{\cancel{4}}{\cancel{4}}=-1$$

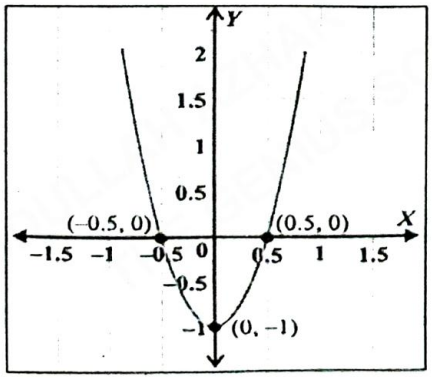
So, we have vertex $(0,-1)$

Step 3: Since $a=4>0$
So, $y\to+\infty$ as $x\to\pm\infty$

Step 4:
Put $x=0$ in (1)

$$y=4\left(0-\frac{1}{2}\right)\left(0+\frac{1}{2}\right)=-\frac{4}{4}=-1$$

The graph cuts the y-axis at $y=-1$



iv) $y=2x^2+x-3$

Sol: $y=2x^2+x-3$

$$y=2x^2+3x-2x-3$$

$$y=x(2x+3)-1(2x+3)$$

$$y=(2x+3)(x-1)$$

$$y=2\left(x+\frac{3}{2}\right)(x-1) \quad \text{..... (1)}$$

Step 1:
x-intercept
Put $y=0$ in (1)

$$2\left(x+\frac{3}{2}\right)(x-1)=0$$

$$\left(x+\frac{3}{2}\right)(x-1)=0\,,2\neq 0$$

$$x+\frac{3}{2}=0 \Rightarrow x=\frac{-3}{2}$$

$$x-1=0 \Rightarrow x=1$$

The graph cuts x-axis at $x=\frac{-3}{2}$ *and* $x=1$

Step 2:

Put $x=\frac{\frac{-3}{2}+1}{2}=\frac{\frac{1}{2}}{2}=\frac{-1}{4}$ in (1)

$$y=2\left(\frac{-1}{4}+\frac{3}{2}\right)\left(\frac{-1}{4}-1\right)$$

$$y=2\left(\frac{-1+6}{4}\right)\left(\frac{-1-4}{4}\right)$$

$$y=\cancel{2}\left(\frac{5}{\cancel{4}_2}\right)\left(\frac{-5}{4}\right)=\frac{-25}{8}$$

So we have vertex $\left(\frac{-1}{4},\frac{-25}{8}\right)$

Step 3:
Since $a=2>0$
So, $y\to+\infty$ as $x\to\pm\infty$

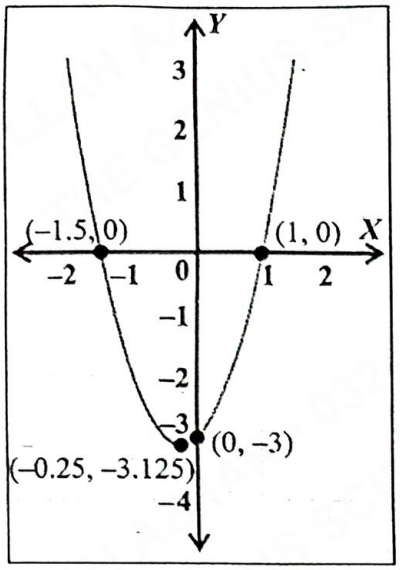
Step 4:

y-intercept
Put $x=0$ in (1)

$$y=2\left(0+\frac{3}{2}\right)(0-1)\Rightarrow y=\cancel{2}\left(\frac{3}{\cancel{2}}\right)(-1)$$

$$y=-3$$

The graph cuts y-axis at $y=-3$



v) $f(x)=x^2+2x+1$

Sol: $y=x^2+2x+1$

$y=(x+1)^2$ ..... (1)

Step 1:

x-intercept

Put $y=0$ in ..... (1)

$(x+1)^2=0$

$x+1=0 \Rightarrow \boxed{x=-1}$

The graph cuts x-axis at $x=-1$

So, we have vertex $(-1,0)$

Step 2:

Since $a=1>0$

So, $y\to+\infty$ as $x\to\pm\infty$

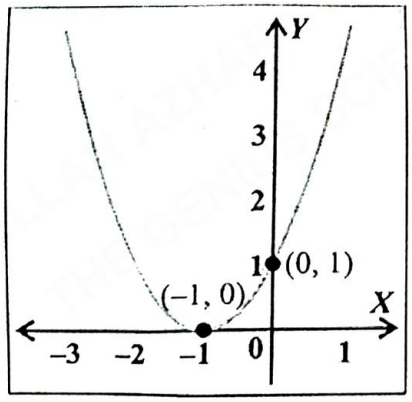
Step 3:

y-intercept

Put $x=0$ in (1)

$y=(0+1)^2 \Rightarrow y=1$

The graph cuts y-axis at $y=1$



vi) $y=3(x-1)^2$

Sol: x-intercept

Put $y=0$ in (1)

$3(x-1)^2=0$

$(x-1)^2=0, 3\neq 0$

$x-1=0 \Rightarrow \boxed{x=1}$

The graph cuts x-axis at $x=1$

So, we have vertex $(1,0)$

Step 2:

Since, $a=3>0$

So, $y\to+\infty$ as $x\to\pm\infty$

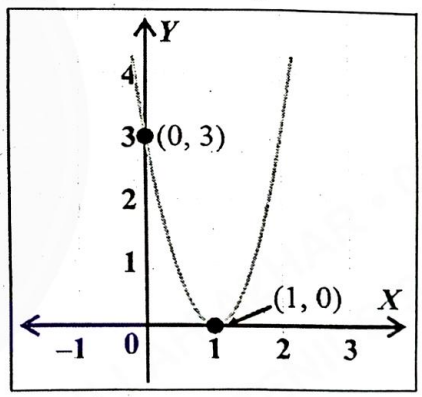
Step 3:

y-intercept

Put $x=0$ in (1)

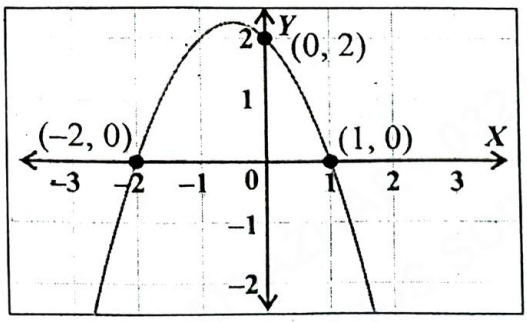
$y=3(0-1)^2 \Rightarrow \boxed{y=3}$

The graph cuts y-axis at $y=3$



2. Find the functions whose graphs are shown in the following figures.

i)



Sol: It is clear from fig.

x-intercepts of the graph are $h=-2,\ k=1$

Use, $y=a(x-h)(x-k)$

$y=a(x+2)(x-1)$ ..... (1)

Since graph passing through point $(0,2)$ so,

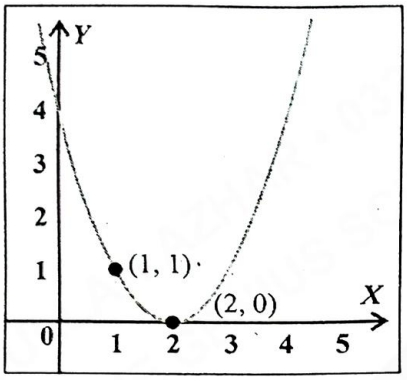
$(1) \Rightarrow 2 = a(0+2)(0-1) \Rightarrow 2 = -2a \Rightarrow a = -1$

$\boxed{a=-1}$ Put in (1)

$y = -1(x+2)(x-1) = -x^2 - x + 2$

Is required function.

ii)



Sol: It is clear from fig.

x-intercept is $h = 2$

Use $y = a(x-h)^2$

$y = a(x-2)^2$ .....(1)

The graph passes through $(1,1)$, So

$1 = a(1-2)^2 \Rightarrow 1 = a(-1)^2 \Rightarrow 1 = a.1$

$\boxed{a=1}$ put in (1)

$y = 1(x-2)^2$

$y = (x-2)^2 = x^2 - 4x + 4$ is required function.

---

3. Find an equation of the graph of a parabola whose x-intercepts are $x = -1$ and $x = -2$ which passes through the point $(-3,2)$.

Sol: Since x-intercepts are $x = -1$ and $x = -2$

So, $h = -2$ and $k = -1$

Use $y = a(x-h)(x-k)$

$y = a(x+2)(x+1)$ .....(1)

The graph passes through point $(-3,2)$

$(1) \Rightarrow 2 = a(-3+2)(-3+1)$

$2 = a(-1)(-2)$

$2 = 2a \Rightarrow \boxed{a=1}$ put in (1)

$(1) \Rightarrow y = 1(x+2)(x+1) = x^2 + 3x + 2$

is required equation.

---

4. Find an equation of the graph of a parabola which touches x-axis at $x = -1$ and cuts y-axis at $y = -2$.

Sol: Since x-intercept is $x = -1$

So, $h = -1$

Use $y = a(x-h)^2$

$y = a(x+1)^2$ ..... (1)

Since graph cuts y-axis at $y = -2$

So, it passes through point $(0,-2)$

$(1) \Rightarrow -2 = a(0+1)^2$

$-2 = a$

Or $a = -2$ put in (1)

$(1) \Rightarrow y = -2(x+1)^2 = -2(x^2 + 2x + 1)$

$= -2x^2 - 4x - 2$ is required equation.

---

5. Find an equation of a quadratic function whose graph cuts x-axis at $x = -1$ and $x = 1$, and whose vertex is at $(0,-2)$.

Sol: Since graph cuts x-axis at $x = -1$ and $x = 1$

(i.e x-intercepts are $x = -1$ and $x = 1$

So, $h = -1$, $k = 1$

Use $y = a(x-h)(x-k)$

$y = a(x+1)(x-1)$ .....(1)

Since graph passes through $(0,-2)$.

$-2 = a(0+1)(0-1)$

$-2 = a(1)(-1) \Rightarrow -2 = -a$

$a = 2$ put in ..... (1)

$(1) \Rightarrow y = 2(x+1)(x-1)$

$y = 2(x^2 - 1) = 2x^2 - 2$

is required equation.

Exercise 1.1 MULTIPLE CHOICE QUESTIONS

1. Vertex of the parabolic curve $y=(x+1)(x-3)$ is:
   A. $(3,0)$ B. $(-1,0)$ C. $(1,-4)$ D. $(0,-3)$
2. The vertex of the quadratic function $f(x)=2x^2-8x+3$ is:
   A. $(2,-5)$ B. $(4,-3)$ C. $(-2,3)$ D. $(2,5)$
3. The graph of $y=2(x-1)(x-3)$ meets the x-axis at
   A. $(1,0)$ B. $(3,0)$ C. both D. None
4. y-intercept of $y=-4x^2-8x-4$ is
   A. $(-1,0)$ B. $(0,-4)$ C. $(1,0)$ D. $(-4,0)$
5. The graph of a quadratic function is called a is
   A. circle B. Hyperbola C. parabola D. Ellipse
6. The vertex of the graph $y=(x-3)^2+2$ is
   A. $(3,2)$ B. $(2,3)$ C. $(-3,2)$ D. $(0,2)$
7. The axis of symmetry of the parabola $y=(x+1)^2-4$ is
   A. $x=-1$ B. $x=1$ C. $y=-4$ D. $y=1$

Answers:

1	C	2	A	3	C	4	B	5	C	6	A	7	A

Exercise 1.2 IMPORTANT TOPICS & EXAMPLES

1.3 Algebraic and Transcendental Functions:

i) **Algebraic Functions:** A function $y=f(x)$ is **algebraic** if it satisfies a polynomial equation of the form

$$P_n(x)y^n+P_{n-1}(x)y^{n-1}+\ldots+P_1(x)y+P_0(x)=0$$

Where each $P_i(x)$ is a polynomial in x for $i=0,1,2,\ldots,n$ and at least one $P_i(x)$ is not identically zero $i=1,2,3,\ldots,n$.

$f(x)=2x^2-3x, g(x)=\dfrac{x^2+1}{x-3}, x\neq 3, h(x)=(x+1)^{\frac{3}{2}}$ and $k(x)=\sqrt{x+1}$ are examples of algebraic functions.

ii) **Transcendental Functions:** A function is called a Transcendental function if it is not a algebraic.

$y=e^x, y=2^x, y=\ln x, y=\sin x, y=\sin^{-1}x, y=\sin x+\ln x, y=e^{x^2}$ are example of transcendental functions.

Example 5: Given the following functions: $f(x)=\sqrt{x+1}, g(x)=e^x, h(x)=\dfrac{x+1}{x^2-2x+1}, q(x)=\ln 2x, r(x)=\sin x$

Classify each function as algebraic or transcendental.

Sol: The following are algebraic functions:

$$f(x)=\sqrt{x+1},\ h(x)=\frac{x+1}{x^2-2x+1}$$

The following are transcendental function $g(x)=e^x, q(x)=\ln 2x, r(x)=\sin x$

1.4 Exponential Functions:

The function defined by $f(x)=a^x$ for all $x\in\mathbb{R}$, where $a>0$ and $a\neq 1$ is called an **exponential function.** The number a called **base** of the exponential function. The function $f(x)=e^x$ is called the **natural exponential function,** where e is number defined by $e=\sum_n^{\infty}\dfrac{1}{n!}$ and its approximate value is $e\approx 2.71828$.