Exercise 1.1 MULTIPLE CHOICE QUESTIONS

1. Vertex of the parabolic curve $y=(x+1)(x-3)$ is:
   A. $(3,0)$ B. $(-1,0)$ C. $(1,-4)$ D. $(0,-3)$
2. The vertex of the quadratic function $f(x)=2x^2-8x+3$ is:
   A. $(2,-5)$ B. $(4,-3)$ C. $(-2,3)$ D. $(2,5)$
3. The graph of $y=2(x-1)(x-3)$ meets the x-axis at
   A. $(1,0)$ B. $(3,0)$ C. both D. None
4. y-intercept of $y=-4x^2-8x-4$ is
   A. $(-1,0)$ B. $(0,-4)$ C. $(1,0)$ D. $(-4,0)$
5. The graph of a quadratic function is called a is
   A. circle B. Hyperbola C. parabola D. Ellipse
6. The vertex of the graph $y=(x-3)^2+2$ is
   A. $(3,2)$ B. $(2,3)$ C. $(-3,2)$ D. $(0,2)$
7. The axis of symmetry of the parabola $y=(x+1)^2-4$ is
   A. $x=-1$ B. $x=1$ C. $y=-4$ D. $y=1$

Answers:

1	C	2	A	3	C	4	B	5	C	6	A	7	A

Exercise 1.2 IMPORTANT TOPICS & EXAMPLES

1.3 Algebraic and Transcendental Functions:

i) **Algebraic Functions:** A function $y=f(x)$ is **algebraic** if it satisfies a polynomial equation of the form

$$P_n(x)y^n+P_{n-1}(x)y^{n-1}+\ldots+P_1(x)y+P_0(x)=0$$

Where each $P_i(x)$ is a polynomial in x for $i=0,1,2,\ldots,n$ and at least one $P_i(x)$ is not identically zero $i=1,2,3,\ldots,n$.

$f(x)=2x^2-3x, g(x)=\dfrac{x^2+1}{x-3}, x\neq 3, h(x)=(x+1)^{\frac{3}{2}}$ and $k(x)=\sqrt{x+1}$ are examples of algebraic functions.

ii) **Transcendental Functions:** A function is called a Transcendental function if it is not a algebraic.

$y=e^x, y=2^x, y=\ln x, y=\sin x, y=\sin^{-1}x, y=\sin x+\ln x, y=e^{x^2}$ are example of transcendental functions.

Example 5: Given the following functions: $f(x)=\sqrt{x+1}, g(x)=e^x, h(x)=\dfrac{x+1}{x^2-2x+1}, q(x)=\ln 2x, r(x)=\sin x$

Classify each function as algebraic or transcendental.

Sol: The following are algebraic functions:

$$f(x)=\sqrt{x+1},\ h(x)=\frac{x+1}{x^2-2x+1}$$

The following are transcendental function $g(x)=e^x, q(x)=\ln 2x, r(x)=\sin x$

1.4 Exponential Functions:

The function defined by $f(x)=a^x$ for all $x\in\mathbb{R}$, where $a>0$ and $a\neq 1$ is called an **exponential function.** The number a called **base** of the exponential function. The function $f(x)=e^x$ is called the **natural exponential function,** where e number defined by $e=\sum_n^{\infty}\frac{1}{n!}$ and its approximate value is $e\approx 2.71828$.

Strictly Increasing and Strictly Decreasing Functions:

Let $y=f(x)$ be a real-valued function and let $x_1, x_2 \in D_f$ be any two points such that $x_1 < x_2$. The function f is called:

i) **Strictly increasing** if $f(x_1) < f(x_2)$. In this case, the graph of the function rises as *x* moves from left to right along the *x*-axis.

ii) **Strictly decreasing** if $f(x_1) > f(x_2)$. In this case, the graph of the function falls as *x* moves from left to right along the *x*-axis.

For Example: Consider the function $y=x^2$

This functions is strictly decreasing $]-\infty,0[$ and strictly increasing on $]0,-\infty[$.

Example 6: Graph of $f(x)=e^x$

Sol: Let $f(x)=e^x$.

Domain of the function if $R=(-\infty,\infty)$. Range of $f(x)=e^x$ is $(0,\infty)$

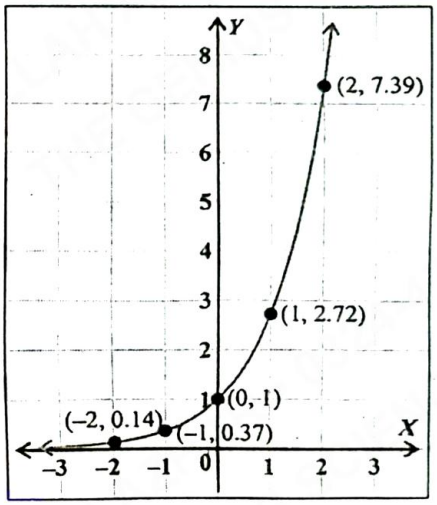
The graph cuts y-axis is $(0,1)$ $y=0$ is the horizontal asymptote.

f is continuous and strictly increasing, because $e>1$.

Some approximate functional values are given in the following tables.

x	$f(x)=e^x$
1	$e=2.718$
2	7.389
3	20.085
10	22026.465
100	2.688×10^{43}
$\to+\infty$	$\to+\infty$

x	$f(x)=e^x$
-1	0.367
-2	0.135
-3	0.0497
-10	0.0000454
-100	3.720×10^{-44}
$\to-\infty$	$\to 0$



Example 7: Graph of $f(x)=\left(\frac{1}{2}\right)^x=\frac{1}{2^x}$

Sol: The graph of the exponential function $f(x)=\left(\frac{1}{2}\right)^x$ has following important properties

Domain of the function is $R=(-\infty,\infty)$ Range of $f(x)=\left(\frac{1}{2}\right)^x$ is $(0,\infty)$.

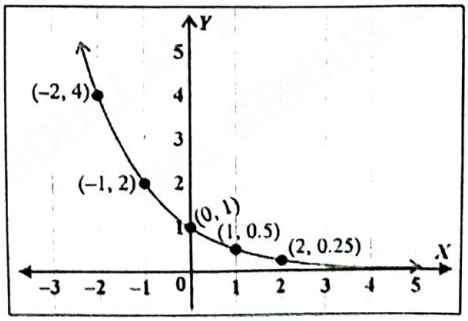
The graph cuts y-axis at $(0,1)$. $y=0$ is the horizontal asymptote.

f is continuous and strictly decreasing because $a=\frac{1}{2}<1$.

Some approximate functional values are given in the following tables.

x	$f(x)=\left(\frac{1}{2}\right)^x$
1	0.5
2	0.25
3	0.125
10	0.0009766
100	7.89×10^{-31}
$\to+\infty$	$\to 0$

x	$f(x)=\left(\frac{1}{2}\right)^x$
-1	2
-2	4
-3	8
-10	1024
-100	1.267×10^{30}
$\to-\infty$	$\to\infty$



HYPERBOLIC FUNCTIONS			
Function	**Definition**	**Domain**	**Range**
$\sinh x$	$\sinh x=\frac{e^x-e^{-x}}{2}$	$R=(-\infty,\infty)$	$R=(-\infty,\infty)$
$\cosh x$	$\cosh x=\frac{e^x+e^{-x}}{2}$	$R=(-\infty,\infty)$	$[1,\infty)$
$\tanh x=\frac{\sinh x}{\cosh x}$	$\tanh x=\frac{e^x-e^{-x}}{e^x+e^{-x}}$	$R=(-\infty,\infty)$	$(-1,1)$
$\coth x=\frac{\cosh x}{\sinh x}$	$\coth x=\frac{e^x+e^{-x}}{e^x-e^{-x}}$	$R\setminus\{0\}$	$(-\infty,-1)\cup(1,\infty)$
$\operatorname{sech} x=\frac{1}{\cosh x}$	$\operatorname{sech} x=\frac{2}{e^x+e^{-x}}$	$R=(-\infty,\infty)$	$(0,1]$
$\operatorname{csch} x=\frac{1}{\sinh x}$	$\operatorname{csch} x=\frac{2}{e^x-e^{-x}}$	$R\setminus\{0\}$	$(-\infty,0)\cup(0,\infty)$

Hyperbolic functions are so named because the parametric equations $x=\cosh t$ and $y=\sinh t$, satisfies the equation of the hyperbola $x^2-y^2=1$.

Example 8: Prove that $\cosh^2 x-\sinh^2 x=1$.

Sol: We know that $\cosh x=\frac{e^x+e^{-x}}{2}$ and $\sinh x=\frac{e^x-e^{-x}}{2}$

Therefore, $\cosh^2 x-\sinh^2 x=\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2=\left(\frac{e^{2x}+e^{-2x}+2}{4}\right)-\left(\frac{e^{2x}+e^{-2x}-2}{4}\right)$

$$\frac{e^{2x}+e^{-2x}+2-e^{2x}-e^{-2x}+2}{4}=\frac{4}{4}=1$$

Hence, $\cosh^2 x-\sin h^2 x=1$

1.5.3 Inverse hyperbolic function

The formulae, domains and ranges of all inverse hyperbolic functions are given in the following table.

Function	Formula	Domain	Range
$\sinh^{-1} x$	$\ln\left(x+\sqrt{x^2+1}\right)$	$(-\infty,\infty)$	$(-\infty,\infty)$
$\cosh^{-1} x$	$\ln\left(x+\sqrt{x^2-1}\right)$	$[1,\infty)$	$[0,\infty)$
$\tanh^{-1} x$	$\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$	$(-1,1)$	$(-\infty,\infty)$
$\coth^{-1} x$	$\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$	$(-\infty,1)\cup(1,\infty)$	$(-\infty,0)\cup(0,\infty)$
$\operatorname{sech}^{-1} x$	$\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$	$(0,1]$	$[0,\infty)$
$\operatorname{csch}^{-1} x$	$\ln\left(\frac{1}{x}+\sqrt{\frac{1}{x^2}+1}\right)$	$(-\infty,0)\cup(0,\infty)$	$(-\infty,0)\cup(0,\infty)$

Example 9: Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$.

Sol: Let $y = \sinh^{-1} x$, where $x, y \in (-\infty, \infty)$.

$$x = \sinh y \Rightarrow x = \frac{e^y - e^{-y}}{2} \Rightarrow 2x = e^y - \frac{1}{e^y}$$

$$\Rightarrow 2x = \frac{e^{2y}-1}{e^y} \Rightarrow 2xe^y = e^{2y} - 1$$

$$\Rightarrow \left(e^y\right)^2 - 2xe^y - 1 = 0$$

Using quadratic formula:

$$e^y = \frac{2x \pm \sqrt{4x^2+4}}{2} = x \pm \sqrt{x^2+1}$$

Note that, $x - \sqrt{x^2+1} < 0 \forall x \in (-\infty, \infty)$.

since $e^y > 0$, we take the positive sign:

$$e^y = x + \sqrt{x^2+1} \Rightarrow y = \ln\left(x + \sqrt{x^2+1}\right)$$

Hence, $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$

Important
if $y = f(x)$ is a bijective function,
then $x = f^{-1}(y)$ is its inverse.

Example 10: Prove that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$ for all $x \in [1, \infty)$.

Sol: For $y \in [0, \infty)$ = Restricted Dom of $\cosh x$ and $x \in [1, \infty)$ = Range of $\cosh x$.

$y = \cosh^{-1} x$; $x = \cosh y$

$$x = \frac{e^y + e^{-y}}{2} \Rightarrow 2x = e^y + \frac{1}{e^y}$$

$$\Rightarrow 2x = \frac{e^{2y}+1}{e^y} \Rightarrow 2xe^y = e^{2y} + 1$$

$$\Rightarrow \left(e^y\right)^2 - 2xe^y + 1 = 0$$

Using quadratic formula:

$$e^y = \frac{2x \pm \sqrt{4x^2-4}}{2}$$

$\Rightarrow e^y = x \pm \sqrt{x^2-1}$, where $x \geq 1$, $y \geq 0$ and $e^y > 0$.

$y = \ln\left(x \pm \sqrt{x^2-1}\right)$.

Is can easily be observed that $0 < x - \sqrt{x^2-1} < 1$ for all $x \geq 1$.

Thus, $y = \ln\left(x - \sqrt{x^2-1}\right) < 0$.

But $y \geq 0$, which is possible only if the positive sign is taken.

Hence, $y = \ln\left(x + \sqrt{x^2-1}\right) \Rightarrow \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$

Important
$Dom f = Range\, f^{-1}$
$Range\, f = Dom\, f^{-1}$

Exercise 1.2 SOLUTION

1. Which of the following functions are algebraic?

$f(x)=\sec x,\ g(x)=\dfrac{2x-3}{x+1},\ h(x)=\sqrt{x^2+2x}$

$k(x)=\dfrac{e^x-e^{-x}}{2},\ r(x)=(x-1)^3$

Sol:

$g(x)=\dfrac{2x-3}{x+1}$	Algebraic function
$h(x)=\sqrt{x^2+2x}$	Algebraic function
$r(x)=(x-1)^3$	Algebraic function
$f(x)=\sec x$	Transcendental function
$k(x)=\dfrac{e^x-e^{-x}}{2}$	Transcendental function

2. Which of the following functions are transcendental.

$f(x)=\dfrac{1}{x},\ g(x)=\cot x, h(x)=e^{2x}+\sqrt{x^2+2x}$

$k(x)=\dfrac{e^x+e^{-x}}{2},\ r(x)=(x-1)^{\frac{3}{2}}$

Sol:

$f(x)=\dfrac{1}{x}$	Algebraic function
$g(x)=\cot x$	Transcendental function
$h(x)=e^{2x}+\sqrt{x^2+2x}$	Transcendental function
$k(x)=\dfrac{e^x-e^{-x}}{2}$	Transcendental function
$r(x)=(x-1)^{\frac{-3}{2}}$	Algebraic function

3. Discuss domain, range and graph of the function $f(x)=3^x$.

Sol: Domain $=(-\infty,+\infty)$

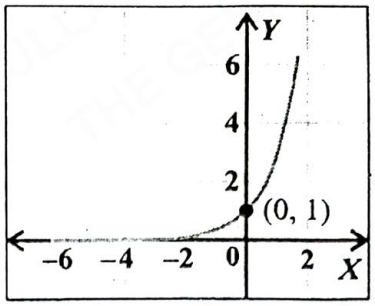
Range $=(0,+\infty)$

The graph cuts y-axis at $(0,1)$

$y=0$ is the horizontal asymptote.

f is the continuous and strictly increasing because $3>1$

$-\infty$ ←	-3	-2	-1	0	1	2	3	→ $+\infty$
0 ←	0.037	0.111	0.333	1	3	9	27	→ $+\infty$



4. Discuss domain, range and graph of the func

$f(x)=\left(\dfrac{1}{3}\right)^x$.

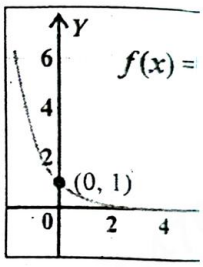
Sol: Domain $=(-\infty,+\infty)$

Range $=(0,+\infty)$

The graph cuts y-axis at $(0,1)$

$y=0$ is the horizontal asymptotes.

f is the continuous and strictly decreasing because $\dfrac{1}{3}<0$



$-\infty$ ←	−3	−2	-1	0	1	2	3
$+\infty$ ←	27	9	3	1	0.333	0.111	0.037

5. Prove the following hyperbolic identities:

i) $1-\tanh^2 x=\operatorname{sech}^2 x$ **ii)** $\operatorname{csch}^2 x=\coth^2$

iii) $\sinh 2x=2\sinh x\ cohx$

iv) $\cosh 2x=\cosh^2 x+\sinh^2 x$

i) $1-\tan h^2 x=\operatorname{sech}^2 x$

Sol: $1-\tan h^2 x=\operatorname{sech}^2 x$

L.H.S $=1-\tan h^2 x$

$=1-\left(\dfrac{e^x-e^{-x}}{e^x+e^{-x}}\right)^2 \quad \because \tan hx=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}$

$=1-\dfrac{\left(e^x-e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}=\dfrac{\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2}{\left(e^x+e^{-x}\right)^2}$

$=\dfrac{e^{2x}+e^{-2x}+2-\left(e^{2x}+e^{-2x}-2\right)}{\left(e^x+e^{-x}\right)^2}$

$=\dfrac{e^{2x}+e^{-2x}+2-e^{2x}-e^{-2x}+2}{\left(e^x+e^{-x}\right)^2}=\dfrac{4}{\left(e^x+e^{-x}\right)^2}$

$$=\left(\frac{2}{e^x+e^{-x}}\right)^2=\sec h^2x \qquad \because \sec hx=\frac{2}{e^x+e^{-x}}$$

= R.H.S

ii) $\cos ech^2x=\cot h^2x-1$

Sol: $\cos ech^2x=\cot h^2x-1$

R.H.S $=\cot h^2x-1$

$$=\left(\frac{e^x+e^{-x}}{e^x-e^{-x}}\right)^2-1 \qquad \because \cot hx=\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$=\frac{\left(e^x+e^{-x}\right)^2}{\left(e^x-e^{-x}\right)^2}-1=\frac{\left(e^x+e^{-x}\right)^2-\left(e^x-e^{-x}\right)^2}{\left(e^x-e^{-x}\right)^2}$$

$$=\frac{e^{2x}+e^{-2x}+2-\left(e^{2x}+e^{-2x}-2\right)}{\left(e^x-e^{-x}\right)^2}$$

$$=\frac{\cancel{e^{2x}}+\cancel{e^{-2x}}+2-\cancel{e^{2x}}-\cancel{e^{-2x}}+2}{\left(e^x-e^{-x}\right)^2}=\frac{4}{\left(e^x-e^{-x}\right)^2}$$

$$=\left(\frac{2}{e^x-e^{-x}}\right)^2=\cos ech^2x \qquad \because \cos echx=\frac{2}{e^x-e^{-x}}$$

= L.H.S

iii) $\sin h2x=2\sin hx\cos hx$

Sol: $\sin h2x=2\sin hx\cos hx$

R.H.S $=2\sin hx\cos hx$

$$=\cancel{2}\cdot\frac{e^x-e^{-x}}{\cancel{2}}\cdot\frac{e^x+e^{-x}}{2} \qquad \because \sin hx=\frac{e^x-e^{-x}}{2}$$

$$=\frac{\left(e^x\right)^2-\left(e^{-x}\right)^2}{2} \qquad \because \cos hx=\frac{e^x+e^{-x}}{2}$$

$$=\frac{e^{2x}-e^{-2x}}{2}=\sin h2x \quad \because \sin h2x=\frac{e^{2x}-e^{-2x}}{2}$$

= L.H.S

iv) $\cos h2x=\cos h^2x+\sin h^2x$

Sol: $\cos h2x=\cos h^2x+\sin h^2x$

R.H.S $=\cos h^2x+\sin h^2x$

$$=\left(\frac{e^x+e^{-x}}{2}\right)^2+\left(\frac{e^x-e^{-x}}{2}\right)^2 \qquad \because \cos hx=\frac{e^x+e^{-x}}{2}$$

$$=\frac{\left(e^x+e^{-x}\right)^2}{4}+\frac{\left(e^x-e^{-x}\right)^2}{4} \qquad \because \sin hx=\frac{e^x-e^{-x}}{2}$$

$$=\frac{e^{2x}+e^{-2x}+\cancel{2}+e^{2x}+e^{-2x}-\cancel{2}}{4}$$

$$=\frac{2e^{2x}+2e^{-2x}}{4} \qquad \because \cos h2x=\frac{e^{2x}+e^{-2x}}{2}$$

$$=\frac{\cancel{2}\left(e^{2x}+e^{-2x}\right)}{\cancel{4}^2}=\frac{e^{2x}+e^{-2x}}{2}=\cos h2x$$

= L.H.S

6. Prove the following inverse hyperbolic formulae.

i) $\tan h^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

Sol: $\tan h^{-1}x=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

Let $y=\tan h^{-1}x \qquad \rightarrow$ (i)

where $x\in(-1,1) \quad , \quad y\in(-\infty,+\infty)$

$\tan hy=x$

$$\frac{e^y-e^{-y}}{e^y+e^{-y}}=x \qquad \because \tan hx=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\frac{e^y-\frac{1}{e^y}}{e^y+\frac{1}{e^y}}=x \Rightarrow \frac{\frac{e^{2y}-1}{\cancel{e^y}}}{\frac{e^{2y}+1}{\cancel{e^y}}}=x$$

$$e^{2y}-1=x\left(e^{2y}+1\right)\Rightarrow e^{2y}-1=xe^{2y}+x$$

$$e^{2y}-xe^{2y}=1+x\Rightarrow\left(1-x\right)e^{2y}=1+x$$

$$e^{2y}=\frac{1+x}{1-x}$$

$$2y=\ln\left(\frac{1+x}{1-x}\right)\Rightarrow y=\frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),x\neq 1$$

$$\tan h^{-1}x=\ln\left(\frac{1+x}{1-x}\right) \quad \text{by} \quad \text{(i)}$$

ii) $\cot h^{-1}x=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

Sol: $\cot h^{-1}x=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

Let $y=\cot h^{-1}x \rightarrow$ (i)

Where $x\in R-[-1,1] \quad , \quad y\in R-\{0\}$

$\cot hy=x$

$$\frac{e^y+e^{-y}}{e^y-e^{-y}}=x \qquad \because \cot hx=\frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$\frac{e^y+\frac{1}{e^y}}{e^y-\frac{1}{e^y}}=x \quad \Rightarrow \frac{\frac{e^{2y}+1}{\cancel{e^y}}}{\frac{e^{2y}-1}{\cancel{e^y}}}=x$$

$$e^{2y}+1=x\left(e^{2y}-1\right)\Rightarrow e^{2y}+1=xe^{2y}-x$$

$$x+1=e^{2y}-xe^{2y}\Rightarrow x+1=\left(x-1\right)e^{2y}$$

$$e^{2y}=\frac{x+1}{x-1}$$

$$2y=\ln\left(\frac{x+1}{x-1}\right)\Rightarrow y=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right),x\neq 1$$

$$\cot h^{-1}x=\frac{1}{2}\ln\left(\frac{x+1}{x-1}\right) \text{ by (i)}$$

iii) $\sec h^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

Sol: $\sec h^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

Let $y = \sec h^{-1}x \rightarrow$ (i)

where $x \in (0,1)$, $y \in (0,\infty)$

$\sec hy = x$

$$\frac{2}{e^y + e^{-y}} = x \Rightarrow \frac{2}{e^y + \frac{1}{e^y}} = x$$

$$\frac{2}{\frac{e^{2y}+1}{e^y}} = x \Rightarrow \frac{2e^y}{e^{2y}+1} = x$$

$$2e^y = x\left(e^{2y}+1\right) \Rightarrow 2e^y = xe^{2y} + x$$

$$xe^{2y} - 2e^y + x = 0 \Rightarrow x\left(e^y\right)^2 - 2\left(e^y\right) + x = 0$$

$$e^y = \frac{2 \pm \sqrt{4-4x^2}}{2x} = \frac{2 \pm \sqrt{\left(1-x^2\right)4}}{2x}$$

$$e^y = \frac{2 \pm 2\sqrt{1-x^2}}{2x} = \frac{\cancel{2}\left(1 \pm \sqrt{1-x^2}\right)}{\cancel{2}x}$$

$$e^y = \frac{1 \pm \sqrt{1-x^2}}{x} \Rightarrow e^y = \frac{1+\sqrt{1-x^2}}{x} \quad \because e^y > 0$$

$$y = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$$

$\sec h^{-1}x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right), x \neq 0$ by (i)

iv) $\cos ech^{-1}x = \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right)$

Sol: $\cos ech^{-1}x = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$

Let $y = \cos ech^{-1}x \rightarrow$ (i)

Where $x \in R - \{0\}$, $y \in R - \{0\}$

$\cos echy = x$

$$\frac{2}{e^y - e^{-y}} = x \Rightarrow \frac{2}{e^y - \frac{1}{e^y}} = x$$

$$\frac{2}{\frac{e^{2y}-1}{e^y}} = x \Rightarrow \frac{2e^y}{e^{2y}-1} = x$$

$$2e^y = x\left(e^{2y}-1\right) \Rightarrow 2e^y = xe^{2y} - x$$

$$xe^{2y} - 2e^y - x = 0 \Rightarrow x\left(e^y\right)^2 - 2\left(e^y\right) - x = 0$$

$$e^y = \frac{2 \pm \sqrt{4+4x^2}}{2x} = \frac{2 \pm \sqrt{4\left(1+x^2\right)}}{2x}$$

$$e^y = \frac{2 \pm 2\sqrt{1+x^2}}{2x} = \frac{\cancel{2}\left(1 \pm \sqrt{1+x^2}\right)}{\cancel{2}x}$$

$$e^y = \frac{1 \pm \sqrt{1+x^2}}{x} \Rightarrow e^y = \frac{1+\sqrt{1+x^2}}{x} \quad \because e^y > 0$$

$$y = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right)$$

$\cos ech^{-1}x = \ln\left(\frac{1+\sqrt{1+x^2}}{x}\right), x \neq 0$ By (i)

$= \ln\left(\frac{1}{x} + \sqrt{\frac{1}{x^2}+1}\right)$

Exercise 1.2 MULTIPLE CHOICE QUESTIONS

1. Which of the following are the vertical asymptotes of $f(x) = \frac{1}{x-2}$?

A. $x = 2$ B. $x = -2$ C. $y = 0$ D. $x = \infty$

2. The graph of $y = 3^x$ passes through:

A. (0, 0) B. (1, 1) C. (0, 1 D. (–1, 3)

3. If $\log_b 81 = 4$, then what is the value of *b*?

A. 3 B. 2 C. 9 D. 4

4. What is the inverse function of the exponential function $y = e^x$?

A. $y = \ln x$ B. $y = \log x$ C. $y = 10^x$ D. $y = x^e$

5. Which of the following properties represents graph of $f(x) = e^x$?

A. Symmetric about *y*-axis B. Symmetric about *x*-axis C. Increasing and one-one D. Decreasing and even

6. **Which one of the following is an Algebraic function?**

A. $f(x)=\sqrt{x+1}$ B. $f(x)=\ln 2x$ C. $f(x)=\sin x$ D. $f(x)=e^{x^2}$

7. **Which one of the following is an Transcendental function?**

A. $f(x)=2x^2-3x$ B. $f(x)=(x-1)^3$ C. $f(x)=\cot x$ D. $f(x)=\sqrt{x^2+2x}$

8. **$y=\sin x+\ln x$ is a/an:**

A. Algebraic function B. Fundamental Transcendental function
C. Non-Fundamental Transcendental function D. None

9. **For and exponential function $y=a^x$, which one is necessary?**

A. $x\in\mathbb{R}$ B. $a>0$ C. $a\neq 1$ D. All

10. **Which function represent exponential decay?**

A. $f(x)=e^x$ B. $g(x)=2^x$ C. $h(x)=\left(\frac{1}{2}\right)^x$ D. $g(x)=\left(\frac{1}{2}\right)$

11. **x-intercept of $y=a^x$, is**

A. $(0,1)$ B. $(1,0)$ C. no x-intercept D. $x=0$

12. **Horizontal asymptote of exponential function $y=a^x$, for $a>1$ is**

A. $x=0$ B. $y=0$ C. $(0,\infty)$ D. $(0,0)$

13. **Range of function $y=e^x$ is**

A. $(-\infty,\infty)$ B. $(-\infty,0)$ C. $(0,\infty)$ D. $(-\infty,-1)\cup(1,\infty)$

Answers:

1	A	2	C	3	A	4	A	5	C	6	A	7	C	8	C	9	D	10	C
11	C	12	B	13	C														

Exercise 1.3 IMPORTANT TOPICS & EXAMPLES

1.6 Logarithmic Function

A logarithmic function is defined by $y=\log_a x,\ a>0, a\neq 1, x>0$. It assigns to each positive real number x a unique real number y, such that $a^y=x$.

Logarithmic Function as Inverse of Exponential Function

We know that the exponential function $y=a^x,(a>0,a\neq 1)$ is a one-to-one function with domain $]-\infty,\infty[$ and range $]0,\infty[$. Thus, it is invertible. Interchanging x and y, we obtain: $x=a^y \Rightarrow y=\log_a x$

Hence, the logarithmic function is the inverse of the exponential function with domain $]0,\infty[$ and range $]-\infty,\infty[$.

1.6.2 Graph of Logarithmic Function

The graph of the logarithmic function $y=\log_a x$ is constructed using the properties discussed in the above subsections. In particular, the domain $x>0$ the vertical asymptote $x=0$, the intercept $(1,0)$ the continuity, the injectivity and the monotonic behaviour depending on the value of a together determine the shape and position of the curve. Using these features, the graph can be drawn accurately for different values of the base a.

Figure 1.10 shows the graphs of logarithmic functions with bases greater than 1, whereas
Figure 1.11 represents the graphs of logarithmic functions with bases between 0 and 1.