6. **Which one of the following is an Algebraic function?**

A. $f(x)=\sqrt{x+1}$ B. $f(x)=\ln 2x$ C. $f(x)=\sin x$ D. $f(x)=e^{x^2}$

7. **Which one of the following is an Transcendental function?**

A. $f(x)=2x^2-3x$ B. $f(x)=(x-1)^3$ C. $f(x)=\cot x$ D. $f(x)=\sqrt{x^2+2x}$

8. **$y=\sin x+\ln x$ is a/an:**

A. Algebraic function B. Fundamental Transcendental function
C. Non-Fundamental Transcendental function D. None

9. **For and exponential function $y=a^x$, which one is necessary?**

A. $x\in\mathbb{R}$ B. $a>0$ C. $a\neq 1$ D. All

10. **Which function represent exponential decay?**

A. $f(x)=e^x$ B. $g(x)=2^x$ C. $h(x)=\left(\frac{1}{2}\right)^x$ D. $g(x)=\left(\frac{1}{2}\right)$

11. **x-intercept of $y=a^x$, is**

A. $(0,1)$ B. $(1,0)$ C. no x-intercept D. $x=0$

12. **Horizontal asymptote of exponential function $y=a^x$, for $a>1$ is**

A. $x=0$ B. $y=0$ C. $(0,\infty)$ D. $(0,0)$

13. **Range of function $y=e^x$ is**

A. $(-\infty,\infty)$ B. $(-\infty,0)$ C. $(0,\infty)$ D. $(-\infty,-1)\cup(1,\infty)$

Answers:

1	A	2	C	3	A	4	A	5	C	6	A	7	C	8	C	9	D	10	C
11	C	12	B	13	C														

Exercise 1.3 IMPORTANT TOPICS & EXAMPLES

1.6 Logarithmic Function

A logarithmic function is defined by $y=\log_a x,\ a>0, a\neq 1, x>0$. It assigns to each positive real number x a unique real number y, such that $a^y=x$.

Logarithmic Function as Inverse of Exponential Function

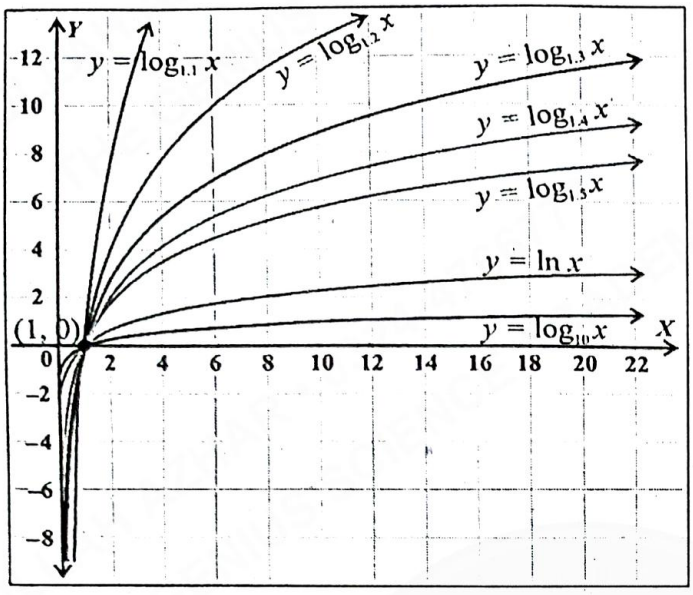
We know that the exponential function $y=a^x,(a>0,a\neq 1)$ is a one-to-one function with domain $]-\infty,\infty[$ and range $]0,\infty[$. Thus, it is invertible. Interchanging x and y, we obtain: $x=a^y\Rightarrow y=\log_a x$

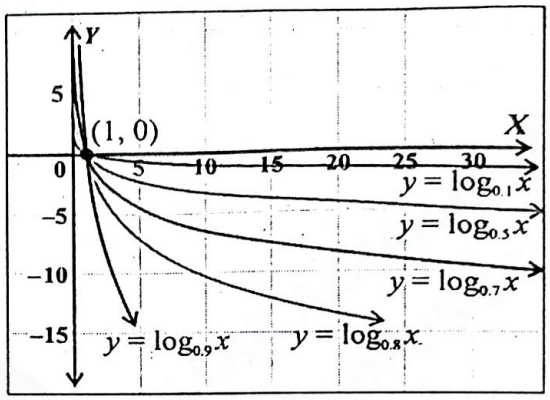
Hence, the logarithmic function is the inverse of the exponential function with domain $]0,\infty[$ and range $]-\infty,\infty[$.

1.6.2 Graph of Logarithmic Function

The graph of the logarithmic function $y=\log_a x$ is constructed using the properties discussed in the above subsections. In particular, the domain $x>0$ the vertical asymptote $x=0$, the intercept $(1,0)$ the continuity, the injectivity and the monotonic behaviour depending on the value of a together determine the shape and position of the curve. Using these features, the graph can be drawn accurately for different values of the base a.

Figure 1.10 shows the graphs of logarithmic functions with bases greater than 1, whereas
Figure 1.11 represents the graphs of logarithmic functions with bases between 0 and 1.





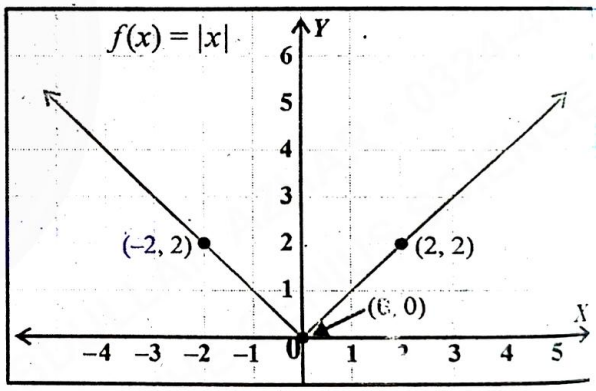
1.7 Modulus Function and its Graph

The modulus function or absolute value function is denoted and defined by:

$$f(x) = |x| = \begin{cases} x, & 0 \\ -x & x > 0 \end{cases}$$

For example, if $x = 2 > 0$ then $|x| = |2| = 2$ and if $x = -2 < 0$ then $|x| = |-2| = -(-2) = 2$.

Similarly, $x = 0$ then $|x| = |0| = 0$. Thus, the modulus of a number is always non-negative. The graph of the modulus function is V-shaped. It consists of two straight lines meeting at the origin. For $x \geq 0$, the graph is the line $y = x$, while for $x < 0$, the graph is the line $y = -x$. thus, the graph of modulus function can be sketched using the points $(0,0), (2,2)$ and $(-2,2)$ as shown in fig.



Example 11: (Logarithmic Graph from Exponential Graph)

Show graphically that the function $y = \ln x$ is the inverse of the function $y = e^x$.

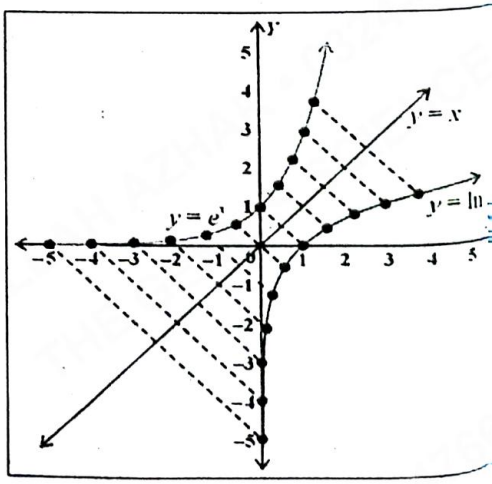
Sol: Consider the exponential $y = e^x$. Note that the function $y = e^x$ is one to one on $]-\infty, \infty[$ and its range is $[0, \infty[$. Hence, it is invertible and its inverse is defined on $]0, \infty[$. We draw the graph of $y = e^x$ along with the line $y = x$ (see fig.).

To obtain the graph of the inverse function, points on the curve $y = e^x$ are reflected about the line $y = x$. The curve passing through these reflected points is then drawn (see fig.). The resulting curve represents the function:

$$y = \ln x$$

Which is clearly defined on the interval $]0, \infty[$.

Hence, the function $y = \ln x$ is the inverse of $y = e^x$.



Example 12: Let $f(x)=\ln(x^2+1)$. Find the function obtained after:

i) a vertical stretch by factor 2 ii) then a shift 1 unit to the left.

Sol: We have: $f(x)=\ln(x^2+1)$.

i) Formula for the vertical scaling is $y=af(x)$

Since, there is a vertical stretch by factor 2, we have $a=2$. Thus, the resulting function becomes:

$$y=2\ln(x^2+1)$$

ii) Now we have $g(x)=2\ln(x^2+1)$.

Formula for the horizontal translation is $y=g(x-h)$.

Since, there is a shift of 1 unit to the left, we have $h=-1$. Thus, the resulting function becomes:

$$y=g(x-(-1))=g(x-h)=2\ln\left[(x+1)^2+1\right]=2\ln(x^2+2x+1+1)$$

Hence, the transformed function is $=2\ln(x^2+2x+2)$

Example 13: Let $f(x)=\dfrac{x}{x+1}$. Find the function obtained after applying all the following transformations with the given order: i) Vertical stretch by factor 2 ii) horizontal stretch by 3 iii) shift 2 units to the right iv) shift 1 unit downward

Sol: Given function: $f(x)=\dfrac{x}{x+1}$.

(i) vertical stretch by 2.

The resulting function is $y=af(x)$ where $a=2$.

Thus, we get:

$$y=2\frac{x}{x+1}=\frac{2x}{x+1}$$

We set, $g(x)=\dfrac{2x}{x+1}$.

(ii) Horizontal stretch by 3.

The resulting function is $y=g(bx)$, where

$b=\dfrac{1}{3}$. So, we have:

$$y=g\left(\frac{x}{3}\right)=\frac{2\frac{x}{3}}{\frac{x}{3}+1}=\frac{2x}{x+3}$$

Let $s(x)=\dfrac{2x}{x+3}$

(iii) shift 2 units to right

The resulting function is $y=(x-h)$, where $h=2$.

So, we get

$$y=s(x-2)=\frac{2(x-2)}{(x-2)+3}=\frac{2x-4}{x+1}$$

Let $t(x)=\dfrac{2x-4}{x+1}$

(iv) Shift 1 unit downward

The resulting function is $y=t(x)+k$, where $k=-1$

Thus we have

$$y=t(x)-1=\frac{2x-4}{x+1}=\frac{x-5}{x+1}$$

Hence, transformed function is $y=\dfrac{x-5}{x+1}$.

Exercise 1.3 SOLUTION

1. Let $f(x) = \log_4(x-2)$

i) Show that $f(x)$ is one-to-one function using horizontal line test.

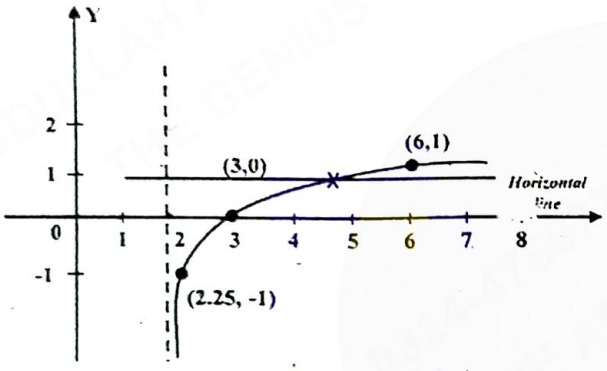
Sol: $f(x) = \log_4(x-2)$ is defined for $x-2>0$

$\Rightarrow x > 2$

First we draw its graph then apply horizontal line test on it

x	2.25	3	6	18
y	−1	0	1	2

It is clear from above figure that any horizontal line crosses the curve of f only one. Which indicates that f is one to one.



ii) Find the inverse function $f^{-1}(x)$ algebraically.

Sol: Since $f(x) = \log_4(x-2)$ Or $y = \log_4(x-2)$

$x - 2 = 4^y$

Interchanging x and y.

$y - 2 = 4^x \Rightarrow y = 4^x + 2$

Put $y = f^{-1}(x)$

$f^{-1}(x) = 4^x + 2$

iii. State the domain and range of $f(x)$ and $f^{-1}(x)$

Sol: Domain f = Range $f^{-1} = (2, \infty)$

Range f = Domain $f^{-1} = (-\infty, \infty)$

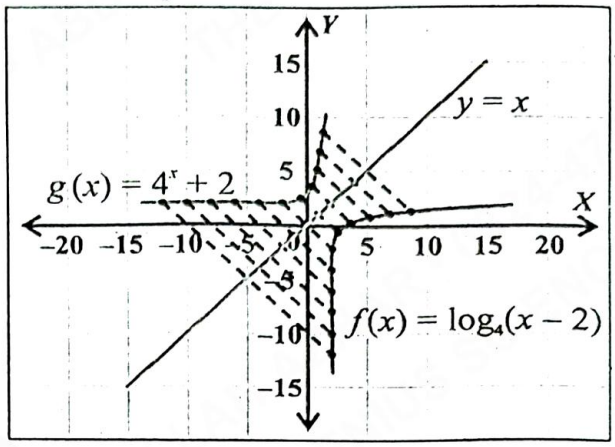
iv) Verify graphically that graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about line $y = x$.

Sol: Domain f = Range $f^{-1} = (2, \infty)$

$y = \log_4(x-2)$	
x	y
2.25	−1
3	0
6	1
18	2

$y = 4^x + 2$	
x	y
−1	2.25
0	3
1	6
2	18

It is clear from figure that $f(x)$ and $f^{-1}(x)$ are symmetric about $y = x$.



2. Let $f(x) = \sin x$ for all $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

i. State the domain and range of $f(x)$ and f^{-1} (

ii. Sketch the graph of $f(x)$ and show that it is one to one using horizontal line test.

iii. Draw the graph of $f^{-1}(x)$ using the reflection of $f(x)$ along the line $y = x$.

Sol i): If $f(x) = \sin x \Rightarrow f^{-1}(x) = \sin^{-1} x$

Domain f = Range $f^{-1} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

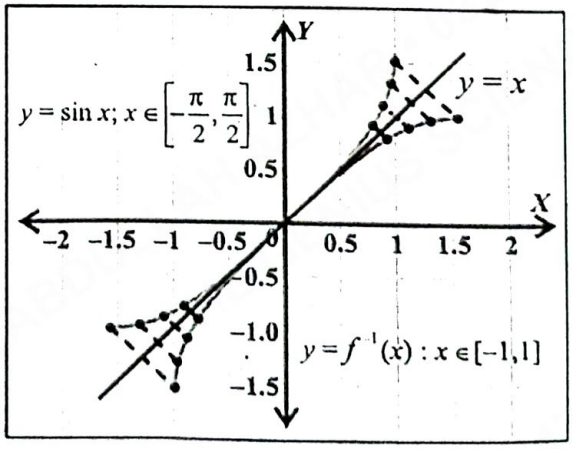
Range f = Domain $f^{-1} = [-1, 1]$

Sol ii) & iii): $y = \sin x$

x	−1.57	−0.52	0	0.52	1.57
y	−1	−0.5	0	0.5	1

$y = \sin^{-1} x$

x	−1	−0.5	0	0.5	1
y	−1.57	−0.52	0	0.52	1.57



Sol ii): It is clear from graph of $f(x) = y = \sin x$ that any horizontal line crosses the curve of f only one. Which indicates that f is one to one.
It is clear from figure that $f(x)$ and $f^{-1}(x)$ are symmetric about $y = x$

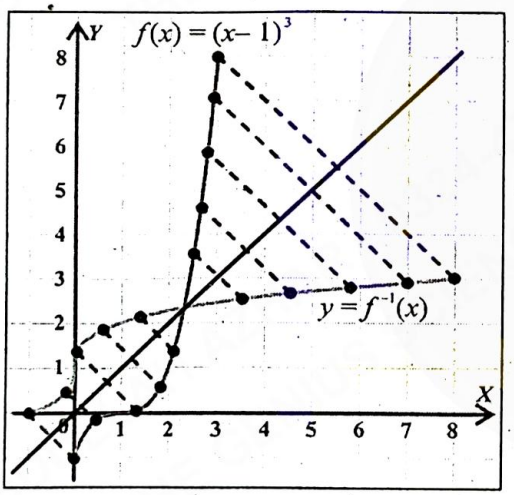
3. A one to one function is defined by $f(x) = (x-1)^3$, $x \in [0,3]$ using the concept of reflection in the line $y = x$ sketch the graph of the inverse function $f^{-1}(x)$

Sol: $y = (x-1)^3, x \in [0,3]$

x	0	1	2	3
y	−1	0	1	8

$x = (y-1)^3$ (just interchanging x and y)

x	−1	0	1	8
y	0	1	2	3



4. Give the function $f(x) = \sqrt{x}$. Find the equation of the function after

i) Vertical stretch by factor 2.

Sol: Given $f(x) = \sqrt{x}$ _____ (1)
For vertical stretch, $y = af(x)$
Here $a = 2$
So, $y = 2f(x)$
$y = 2\sqrt{x}$ by (1)

ii) Horizontal shift 3 units to the right

Sol: For horizontal right shift, we have $y = f(x-h)$
Here $h = 3$
So $y = f(x-3)$
$y = \sqrt{x-3}$ by (1)

iii) Apply both transformations (i) and (ii)

Sol: For vertical stretch, we have $y = af(x)$
Here $a = 2$
So, $y = 2f(x)$
$y = 2\sqrt{x}$ by (1)
Now, $g(x) = 2\sqrt{x}$ _____ (2)
For horizontal right shift, we have $y = g(x-h)$
Here $h = 3$
So $y = g(x-3)$
$y = 2\sqrt{x-3}$ by (2)

5. Given the function $f(x) = \frac{1}{x}, x \neq 0$. Find the equation of the transformed function after applying the following transformations in order. Horizontal shift 2 units to the Left, horizontal compression by factor 2, vertical stretching by factor 3 and vertical shift 1 unit upward.

Sol: Given $f(x) = \frac{1}{x}$ _____ (1)
For horizontal right shift, we have
$y = f(x-h)$
Here $h = -2$
So $y = f(x+2)$
$y = \frac{1}{x+2}$ by (1)
Now, Let $g(x) = \frac{1}{x+2}$ _____ (2)
For horizontal compress, we have
$y = g(ax)$
Here $a = 2$
So $y = g(2x)$
$y = \frac{1}{2x+2}$ by (2)
Now, Let $s(x) = \frac{1}{2x+2}$ _____ (3)
For vertical stretch, we have
$y = as(x)$
Here $a = 3$
So $y = 3s(x)$
$y = \frac{3}{2x+2}$
Now, Let $t(x) = \frac{3}{2x+2}$ _____ (4)
For vertical up stretch, we have

$y = t(x) + k$

Here $k = 1, y = t(x) + 1$

$y = \frac{3}{2x+2} + 1$ by (4)

$y = \frac{3+2x+2}{2x+2} = \frac{2x+5}{2x+2}$

6. Given the function f(x) = ln(x) find the equation of the transformed function obtained after shifting the graph 1 unit to the right, vertically compressing it by a factor of 2, and then shifting it 3 units upward.

Sol: Given $f(x) = \ln x$ _____ (1)

For right shift, we have

$y = f(x-h)$

Here $h = 1$

So $y = f(x-1)$

$y = \ln(x-1)$ by (1)

Now, Let $g(x) = \ln(x-1)$ _____ (2)

For vertical compress,

$y = ag(x)$ $\because 0 < a < 1$, take $a = \frac{1}{b}$

$y = \frac{1}{b} g(x)$

Here $b = 2$

So $y = \frac{1}{2} g(x)$

$y = \frac{1}{2} \ln(x-1)$

Now, Let $s(x) = \frac{1}{2} \ln(x-1)$ _____ (3)

For vertical shift upward, we have $y = s(x) + k$

Here $k = 3$

$y = s(x) + 3 \Rightarrow y = \frac{1}{2} \ln(x-1) + 3$

By using law of logarithmic

$y = \ln(x-1)^{\frac{1}{2}} + 3 \Rightarrow y = \ln\sqrt{x-1} + 3$

7. Given the function $f(x) = \sqrt{\frac{2x-1}{x+3}}$. Find the equation of the transformed function after applying in order, the horizontal shift 2 units to the left, horizontal compression by factor 3, vertical compression by factor 2, and vertical shift 1 unit downward.

Sol: Given $\sqrt{\frac{2x-1}{x+3}}$ _____ (1)

For right shift, we have $y = f(x-h)$

Here $h = -2$

So $y = f(x+2)$

$y = \sqrt{\frac{2(x+2)-1}{x+2+3}} = \sqrt{\frac{2x+4-1}{x+5}}$ by (1)

Now, Let $g(x) = \sqrt{\frac{2x+3}{x+5}}$ _____ (2)

For vertical compress, $y = ag(x)$

Here $a = 3$

So $y = g(3x)$

$y = \sqrt{\frac{2(3x)+3}{3x+5}} = \sqrt{\frac{6x+3}{3x+5}}$

Now, Let $s(x) = \sqrt{\frac{6x+3}{3x+5}}$ _____ (3)

For vertical compress, we have

$y = as(x)$ $\because 0 < a < 1$, take $a = \frac{1}{b}$

$y = \frac{1}{b} s(x)$

Here $b = 2$

So $y = \frac{1}{2} s(x)$

$y = \frac{1}{2}\sqrt{\frac{6x+3}{3x+5}}$

$t(x) = \frac{1}{2}\sqrt{\frac{6x+3}{3x+5}}$ _____ (4)

For vertical shift downward $y = t(x) - k$

Here $k = 1$

$y = \frac{1}{2}\sqrt{\frac{6x+3}{3x+5}} - 1$

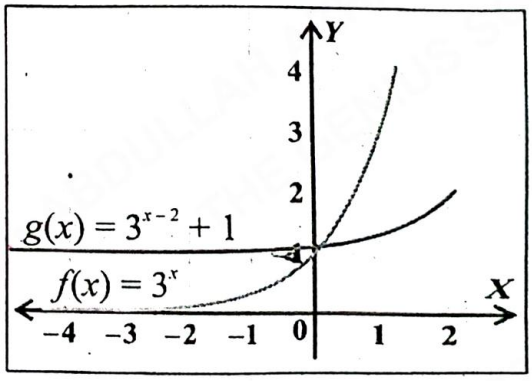
8. Given the function $f(x) = 3^x$ and its transformed function $g(x) = 3^{x-2} + 1$, draw the graphs of both functions on the same coordinate plane and compare their shapes.

Sol: $f(x) = 3^x$

x	-2	-1	0	1	2
y	0.11	0.33	1	3	9

$g(x) = 3^{x-2} + 1$

x	-2	-1	0	1	2	3	4
y	1.01	1.03	1.11	1.33	2	4	10



The function $g(x)=3^{x-2}+1$ is derived from $f(x)=3^x$ by applying two transformations

i. Horizontal right shift

The term $x-2$ is the power of 3 in $g(x)$ shows that graph of $f(x)$ is shifted 2 unit to right.

ii. Vertical up shift

The term $+1$ in $g(x)$ shows that $f(x)$ is shifted upward 1 unit

9. Given the function $f(x)=\ln x$. Find the equation of the transformed function after applying in order, the vertical stretch by factor 4, shift 6 units to right, horizontal compress by factor 3, and shift 1 unit upward. Draw the graphs of both the original and transformed functions on the same coordinate plane.

Sol: Given $f(x)=\ln x$ _____ (1)

For vertical stretch, we have

$y=af(x)$

Here $a=4$

So $y=4f(x)$

$y=4\ln x$ by (1)

Now, Let $g(x)=4\ln x$ _____ (2)

For right shift,

$y=g(x-h)$

Here $h=6$

So $y=g(x-6)$

$y=4\ln(x-6)$

Now, Let $s(x)=4\ln(x-6)$ _____ (3)

For horizontal compress, we have

$y=s(ax)$

Here $a=3$

$y=s(3x)$

$y=4\ln(3x-6)$

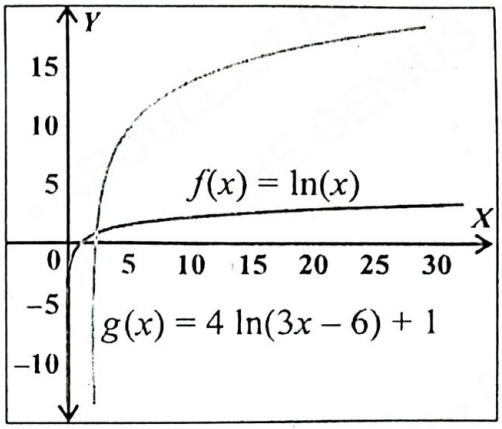
Now, let $t(x)=4\ln(3x-6)$ _____ (4)

For up shift,

$y=t(x)-k$

Here $k=1$

$y=4\ln(3x+6)+1$



$y=\ln x$

x	0.25	0.5	1	2	3	4	5
y	−1.4	−0.7	0	0.7	1.1	1.4	1.6

$y=4\ln(3x+6)+1$

x	1	2	3	4	5
y	9.8	10.9	11.83	12.56	13.17

The function $g(x)=3^{x-2}+1$ is derived from $f(x)=3^x$ by applying two transformations.

10. Draw the graph of $f(x)=|x|$ after applying in order, shift of 7 units to right, horizontal compress by factor 2, vertical shift 6 units downward, and vertical compress by factor 2.

Sol: Given $f(x)=|x|$ _____ (1)

For right shift,

$y=f(x-h)$

Here $h=7$

So $y=f(x-7)$

$y=|x-7|$

Now, Let $g(x)=|x-7|$ _____ (2)

For horizontal compress, we have

$y=g(ax)$

Here $a=2$

$y=g(2x)$

$y=|2x-7|$

Now, let $s(x)=|2x-7|$ _____ (3)

For shift downward,

$y=s(x)-k$

Here $k=6$

$y=s(x)-6$

$y=|2x-7|-6$

Now, let $t(x)=|2x-7|-6$ _____ (4)

For vertical compress, we have

$y = at(x) \quad \because 0 < a < 1$, take $a = \frac{1}{b}$

$y = \frac{1}{b}t(x)$

Here $b = 2$

So $y = \frac{1}{2}t(x) = \frac{1}{2}[|12x-7|-6]$

$y = \frac{1}{2}|2x-7| - \frac{1}{\cancel{2}} \times \cancel{6}^3$

$y = \frac{1}{2}|2x-7| - 3$ _____ (1)

For vertex, Take $2x-7=0 \Rightarrow x = \frac{7}{2} = 3.5$

Put $x = 3.5$ In (1), we have $y = \frac{1}{2}|0| - 3 = -3$

Here vertex $(3.5, -3)$

X-intercept put $y = 0$ in (1)

$\frac{1}{2}|2x-7| - 3 = 0$

$\frac{1}{2}|2x-7| = 3$

$|2x-7| = 6 \Rightarrow 2x-7 = \pm 6$

$2x-7=6$ $\qquad$ $2x-7=-6$

$2x = 6+7$ $\qquad$ $2x = 7-6$

$2x = 13$ $\qquad$ $2x = 1$

$x = \frac{13}{2} = 6.5$ $\qquad$ $x = \frac{1}{2} = 0.5$

Hence, we have, $(0.5, 0)$ and $(6.5, 0)$

Y-intercept put $x = 0$ in (1)

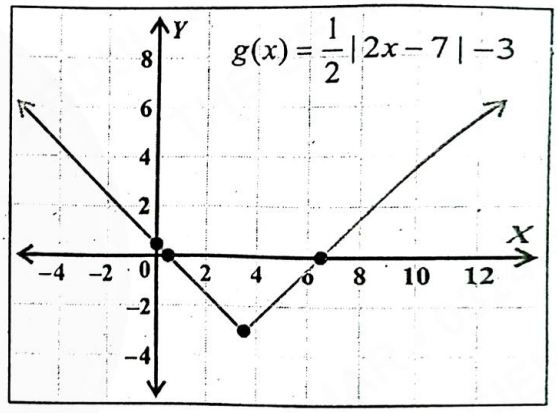
$y = \frac{1}{2}|0-7| - 3$

$y = \frac{1}{2}|-7| - 3$

$y = \frac{7}{2} - 3$

$y = \frac{7-6}{2} = \frac{1}{2} = 0.5$

Hence, we have, $(0.5, 0)$



Exercise 1.3 MULTIPLE CHOICE QUESTIONS

1. What is the domain of $f(x) = \ln(x+4)$?

A. $x \geq -4$ $\qquad$ B. $x > -4$ $\qquad$ C. $x \leq 4$ $\qquad$ D. $x < 4$

2. Which of the following graphs passes the horizontal line test?

A. $y = x^2$ $\qquad$ B. $y = |x|$ $\qquad$ C. $y = \ln x$ $\qquad$ D. $y = \sin x$

3. Which of the following functions is (1, 1) and has inverse?

A. $f(x) = x^3$ $\qquad$ B. $f(x) = x^2$ $\qquad$ C. $f(x) = \cos x$ $\qquad$ D. $f(x) = |x|$

4. If $f(x) = \ln x$, then range of $f^{-1}(x)$ is:

A. $x \neq 0$ $\qquad$ B. $x \in \mathbb{R}$ $\qquad$ C. $x < 0$ $\qquad$ D. $x > 0$

5. What is the range of $f(x) = \tan^{-1} x$?

A. $(0, \pi)$ $\qquad$ B. $[-1, 1]$ $\qquad$ C. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\qquad$ D. $(0, \infty)$

6. If f is one-one and $f(2) = 5$, then value of $f^{-1}(5)$ is:

A. 7 $\qquad$ B. 5 $\qquad$ C. 3 $\qquad$ D. 2

7. Which of the following functions is not one-one and has no inverse over $\mathbb{R}$?

A. $f(x) = e^x$ $\qquad$ B. $f(x) = e^3$ $\qquad$ C. $f(x) = x^2$ $\qquad$ D. $f(x) = \ln x$

8. **The graph of $y=f(x)+3$ is obtained by:**
A. Shift left by 3 B. Shift up by 3 C. Shift down by 3 D. Stretch vertically by 3

9. **If $g(x)=f(x-4)$, the graph of *g* is:**
A. Shifted right 4 units B. Shifted left 4 units C. Shifted up 4 units D. Compressed horizontally

10. **The transformation $y=2f(x)$ causes:**
A. Vertical stretch by 2 B. Horizontal stretch by 2 C. Vertical shift up D. Compression vertically

11. **What does $y=f(3x)$ do to the graph of $f(x)$?**
A. Vertical stretch by 3 B. Horizontal compression by 3 C. Horizontal stretch by 3 D. Shift right by 3

12. **If $y=f(x)-5$, the transformed graph of $f(x)$ shows:**
A. Shifted up by 5 B. Stretched down C. Shifted down by 5 D. Compressed vertically

13. **What type of transformation does graph of $y=\frac{1}{2}f(x)$ show?**
A. Compress vertically by 2 B. Stretch vertically by 2 C. Shift left by 2 D. Shift up by 0.5

14. **What type of transformation graph of $y=f(x+2)$ shows:**
A. Shifted right 2 units B. Shifted left 2 units C. Shifted up 2 units D. Reflected in *y*-axis

15. **The transformation applied on the graph of $y=f(-x)$ causes:**
A. Reflection in *x*-axis B. Reflection in *y*-axis C. Shift left D. Horizontal compression

16. **Which of the following describes $y=-f(x)$?**
A. Reflect in *y*-axis B. Vertical shift down C. Reflect in x-axis D. Horizontal stretch

17. **Which of the following represents a left shift by 4 units of the function $y=\sqrt{x}$?**
A. $y=\sqrt{x+4}$ B. $y=\sqrt{x-4}$ C. $y=\sqrt{4x}$ D. $y=4\sqrt{x}$

18. **The logarithmic function $f(x)=\log_b x$ defined for:**
(a) All real numbers (b) $x<0$ (c) $x>0$ (d) $x\geq 0$

19. **A function $f:A\to B$ has an inverse if and only if**
(a) *f* is one to one (c) *f* is onto
(b) *f* is both one-to-one and onto (d) *f* is neither one-to-one and onto.

20. **The inverse function of $f(x)=x^3$,**
(a) $f^{-1}(x)=x^{-3}$ (b) $f^{-1}(x)=\sqrt{x^{-3}}$ (c) $f^{-1}(x)=\sqrt[3]{x^{-3}}$ (d) $f^{-1}(x)=\sqrt[3]{x}$

21. **Scaling refers to:**
(a) increasing the size of an object
(b) decreasing the size of an object
(c) maintaining the properties while resizing an object
(d) changing the shape of an object

22. **Which of the following statements is true for the uniform scaling?**
(a) Both width and height change proportionally (b) Only the width changes
(c) Only the height changes (d) width and height remain unchanged

23. **What is the effect on the graph of $f(x)$ when it is replaced by $f(x+2)$?**
A) It shifts 2 units to the right B) It shifts 2 units to the left C) It shifts 2 units up D) It shifts 2 units down.

24. **The domain of $y=\sin^{-1}(x)$, is:**
(a) $[0,\infty)$ (b) $(-\infty,\infty)$ (c) $[-1,1]$ (d) $[0,1]$

Answers:

1	B	2	C	3	A	4	D	5	C	6	D	7	C	8	B	9	A	10	A
11	B	12	C	13	A	14	B	15	B	16	C	17	A	18	c	19	c	20	d
21	c	22	a	23	b	24	c												