Exercise 1.5 DEFINITIONS & EXAMPLES

1.11 Applications of Exponential and Logarithmic Functions in Real-Life Situations

i) Growth and Decay

Example 21: (Growth)
A population is given by: $N = 200e^{0.04t}$. Find the time t when the population becomes 400, where t is measured in hours.

Sol: we are given that: $N = 200e^{0.04t}$
Substituting $N = 400$:
$400 = 200e^{0.04t} \Rightarrow 2 = e^{0.04t}$
Take natural logarithm on both sides:
$\ln 2 = 0.04t$
$t = \dfrac{\ln 2}{0.04} \approx 17.33 hours$
Hence the populations become 400 in 17.33 hours approximately.

Example 22: (Decay)
A radioactive substance decays according to $N = 500e^{kt}$ After 20 hours, the amount becomes 300. Find the value of k, where t is in hours.

Sol: we have: $N = 500e^{kt}$
Substituting $t = 20$ and $N = 300$:
$300 = 500e^{20k} \Rightarrow \dfrac{3}{5} = e^{20k}$
$\ln\left(\dfrac{3}{5}\right) = 20k \Rightarrow k = \dfrac{1}{20}\ln\left(\dfrac{3}{5}\right) \approx -0.0256$
Hence $k \approx -0.0256$

Example 23: (Finding sound level)
A sound has intensity $I = 10^{-4}$ and reference intensity $I_0 = 10^{-12}$. Find the sound level.

Sol: We have $L = 10\log\left(\dfrac{I}{I_0}\right)$
Substituting, $I = 10^{-4}$ and $I = 10^{-12}$:
$L = 10\log\left(\dfrac{10^{-4}}{10^{-12}}\right) = 10\log\left(10^{8}\right) = 10 \times 8 = 80dB$

Example 24: A sound has level $L = 60dB$ and $I_0 = 10^{-12}$. Find the intensity I.

Sol: we have: $L = 10\log\left(\dfrac{I}{I_0}\right)$
Substituting $L = 60dB$ and $I_0 = 10^{-12}$:
$60 = 10\log\left(\dfrac{I}{10^{-12}}\right) \Rightarrow 6 = \log\left(\dfrac{I}{10^{-12}}\right)$
Convert to exponential form:
$10^6 = \dfrac{I}{10^{-12}} \Rightarrow I = 10^6 \times 10^{-12} = 10^{-6}\ W/m^2$

Example 25: **The sound levels at two points are L_1 $90dB$ and $L_2 = 60dB$. Find the ratio of their intensities $\dfrac{I_1}{I_2}$, where $I_0 = 10^{-12}\ W/m^2$**

Sol: Given: $L_1 = 90dB$, $L_2 = 60dB$
Formula: $L = 10\log\left(\dfrac{I}{I_0}\right)$
For the given values of L_1 and L_2, we get:
$90 = 10\log\left(\dfrac{I_1}{I_0}\right)$ or $9 = \log\left(\dfrac{I_1}{I_0}\right)$
$60 = 10\log\left(\dfrac{I_2}{I_0}\right)$ or $6 = \log\left(\dfrac{I_2}{I_0}\right)$
Converting in exponential form:
$\dfrac{I_1}{I_0} = 10^9$ and $\dfrac{I_2}{I_0} = 10^6$
Dividing the above equations:
$\dfrac{I_1}{I_2} = \dfrac{10^9}{10^6}$ and $\dfrac{I_1}{I_2} = 10^3 = 1000$
Which is the required ratio of intensities of the sound levels.

Example 26: A sum of Rs. 1000 is invested at a rate of 5% per year compounded continuously for 3 years. Find the final amount.

Sol: Given: $P = 1000$,
$r = 5\% = 0.05 \Rightarrow t = 3$
Formula for the continuous compound interest:
$A = Pe^{rt}$
Substituting the values:
$A = 1000e^{0.05\times 3} = 1000e^{0.15}$
$A \approx 1000 \times 1.1618 = 1161.8$
Hence, the final amount after 3 years is $A \approx Rs.\ 1161.8$

Example 27: A sum of Rs. 5000 is invested at a rate of 8% per annum, compounded quarterly. Find the amount after 3 years.

Sol: we have: $P = 5000$
$r = 0.08 (8\% per\ annum)$
Quarterly compounding $n = 4$
$t = 3 years$
Formula for the discrete compound interest:
$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$
Substituting the values:
$A = 5000\left(1 + \dfrac{0.08}{4}\right)^{4\times 3}$
$A = (5000)(1.02)^{12} \approx 6341.2$
Hence, the amount after 3 years is $A \approx Rs.\ 6341.2$

Exercise 1.5 SOLUTION

1. A population is given by $N = 300e^{0.02t}$, where t is measured in hours. Find the time when the population becomes 600.

Sol. Given, $N = 300e^{0.02t}$

Put $N = 600$

$$600 = 300e^{0.02t} \Rightarrow \frac{600}{300} = e^{0.02t}$$

$$e^{0.02t} = 2 \Rightarrow 0.02t = \ln 2$$

$$t = \frac{\ln 2}{0.02} = 34.65 \text{ hrs.}$$

2. A culture grows according to $N = 150e^{kt}$, where time t is measured in hours. After 10 hours, the population becomes 250. Find the growth constant k

Sol. Given, $t = 10$ hrs.

$N = 250$, $k = ?$

$N = 150e^{kt}$ (given)

Put $t = 10, N = 250$

$$250 = 150 \times e^{10k} \Rightarrow \frac{250}{150} = e^{10k} \Rightarrow \frac{5}{3} = e^{10k}$$

$$\ln\left(\frac{5}{3}\right) = 10k \Rightarrow k = \frac{1}{10}\ln\left(\frac{5}{3}\right) = 0.0511$$

3. A radioactive substance decays as $N = 800e^{-0.05t}$, where t is in hours. Find the time when the amount reduces to 200.

Sol. Given, $N = 200$, $t = ?$

$N = 800e^{-0.05t}$

Put $N = 200$

$$200 = 800e^{-0.05t} \Rightarrow \frac{200}{800} = e^{-0.05t}$$

$$\frac{1}{4} = e^{-0.05t} \Rightarrow \ln\left(\frac{1}{4}\right) = -0.05t$$

$$t = -\frac{\ln(0.25)}{0.05} = 27.73 \text{ hrs.}$$

4. A substance decays according to $N = 600e^{kt}$, where time t is measured in hours. After 8 hours, the amount becomes 400. Find the decay constant k.

Sol. $t = 8$ hrs , $N = 400$, $k = ?$

$N = 600e^{kt}$

Put $t = 8, N = 400$

$$400 = 600e^{8k} \Rightarrow \frac{400}{600} = e^{8k} \Rightarrow \frac{2}{3} = e^{8k}$$

$$\Rightarrow \ln\left(\frac{2}{3}\right) = 8k \Rightarrow k = \frac{1}{8}\ln\left(\frac{2}{3}\right) = -0.0507$$

5. A sound has intensity $I = 10^{-6}\ W/m^2$; reference intensity $I_0 = 10^{-12}\ W/m^2$. Find sound level in decibels.

Sol. Given, $I = 10^{-6}$ W/m^2

$I_0 = 10^{-12}$ W/m^2 , $L = ?$

Use, $L = 10\log\left(\frac{I}{I_0}\right)$

Put $I = 10^{-6}, I_0 = 10^{-12}$

$$L = 10\log\left(\frac{10^{-6}}{10^{-12}}\right) = 10\log\left(10^{-6+12}\right)$$

$$L = 10\log\left(10^6\right) = 10 \times 6\log 10$$

$$L = 60 \times 1 = 60 \text{ dB}$$

6. A sound has level $L = 80\,dB$ and $I_0 = 10^{-12}\ W/r$ Find the intensity I.

Sol. Given, $L = 80$ dB

$I_0 = 10^{-12}$ W/m^2 , $I = ?$

Use, $L = 10\log\left(\frac{I}{I_0}\right)$

Put $L = 80, I_0 = 10^{-12}$

$$80 = 10\log\left(\frac{I}{10^{-12}}\right) \Rightarrow \frac{80}{10} = \log\left(\frac{I}{10^{-12}}\right)$$

$$8 = \log\left(\frac{I}{10^{-12}}\right) \Rightarrow 10^8 = \frac{I}{10^{-12}} \Rightarrow 10^8 \times 10^{-12}$$

$$I = 10^{-12+8} \Rightarrow I = 10^{-4} \text{ W/m}^2$$

7. A quantity changes according to $N = 500e^{kt}$, if $N = 250$ when $t = 5$, find:

i) the value of constant k.

ii) the quantity N when $t = 10$

Sol. Given, $N = 500e^{kt}$, $N = 250$

$t = 5$

i) $k = ?$

$N = 500e^{kt}$

Put $N = 250, t = 5$

$$250 = 500e^{5k} \Rightarrow \frac{250}{500} = e^{5k}$$

$$\frac{1}{2} = e^{5k} \Rightarrow \ln\left(\frac{1}{2}\right) = 5k$$

$$k = \frac{1}{5}\ln\left(\frac{1}{2}\right) = -0.1386$$

ii) $N = ?$
$t = 10$
Use, $N = 500e^{kt}$
Put $t = 10, k = -0.1386$
$N = 500 \times e^{(-0.1386)\times 10} \Rightarrow N = 125.037$
So, $N = 125$

3. **Two sounds have intensities I_1 and I_2 such that $L_1 = 85\,dB$ and $L_2 = 65\,dB$. Find the ratio, $\frac{I_1}{I_2}$, where $I_0 = 10^{-12}\ W/m^2$**

Sol. Given, $L_1 = 85$ dB, $L_2 = 65$ dB
$I_0 = 10^{-12}$ W/m^2 , $\frac{I_1}{I_2} = ?$

Use, $L = 10\log\left(\frac{I}{I_0}\right)$

For L_1,

$L_1 = 10\log\left(\frac{I_1}{I_0}\right)$

$85 = 10\log\left(\frac{I_1}{I_0}\right) \rightarrow (1)$

For L_2,

$L_2 = 10\log\left(\frac{I_2}{I_0}\right)$

$65 = 10\log\left(\frac{I_2}{I_0}\right) \rightarrow (2)$

From (1), $\frac{85}{10} = \log\left(\frac{I_1}{I_0}\right)$

$8.5 = \log\left(\frac{I_1}{I_0}\right) \Rightarrow 10^{8.5} = \frac{I_1}{I_0} \rightarrow (3)$

From (2), $\frac{65}{10} = \log\left(\frac{I_2}{I_0}\right)$

$6.5 = \log\left(\frac{I_2}{I_0}\right) \Rightarrow 10^{6.5} = \frac{I_2}{I_0} \rightarrow (4)$

(3) is divided by (4)

$\frac{10^{8.5}}{10^{6.5}} = \frac{I_1/I_0}{I_2/I_0} \Rightarrow 10^{8.5} \times 10^{-6.5} = \frac{I_1}{I_2}$

$\frac{I_1}{I_2} = 10^{8.5-6.5} = 10^2 = 100$

9. **A sum of Rs. 4000 is invested at 6% per annum, compounded annually for 3 years. Find the final amount.**

Sol. $P = 4000$, $r = 6\% = \frac{6}{100} = 0.06$, $n = 1$
$t = 3$, $A = ?$
We know, $A = P\left(1 + \frac{r}{n}\right)^{nt}$
Put $P = 4000, r = 0.06, n = 1, t = 3$
$A = 4000\left(1 + \frac{0.06}{1}\right)^{1\times 3} = 4000 \times (1.06)^3 = 4764.06$

10. **A sum of Rs. 5000 becomes 6324.48 when invested at 8% per annum, compounded annually. Find the time t.**

Sol. $P = 5000$, $A = 6324.48$
$r = 8\% = 0.08$, $n = 1$, $t = ?$
$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$6324.48 = 5000\left(1 + \frac{0.08}{1}\right)^{1\times t}$
$\frac{6324.48}{5000} = (1.08)^t$
$1.26 = (1.08)^t$ Or $(1.08)^t = 1.26$
$\ln(1.08)^t = \ln(1.26) \Rightarrow t \cdot \ln(1.08) = \ln(1.26)$
$t = \frac{\ln(1.26)}{\ln(1.08)} = 3$ years

11. **A sum of Rs. 3000 is invested at 5% per annum, compounded continuously for 4 years. Find the final amount.**

Sol. $P = 3000$; $r = 5\% = 0.05$; $t = 4$; $A = ?$
$A = Pe^{rt}$
$A = 3000 \times e^{0.05\times 4}$
$A = 3000 \times 1.22$
$A = 3664.2$

12. **An amount of Rs. 2000 becomes 3320.12 under continuous compounding at 6% per annum. Find the time t.**

Sol. Given, $P = 2000$, $A = 3320.12$
$r = 6\% = 0.06$, $t = ?$
$A = Pe^{rt}$
Put $A = 3320.12, P = 2000, r = 0.06$
$3320.12 = 2000 \times e^{0.06t} \Rightarrow \frac{3320.12}{2000} = e^{0.06t}$
$1.66 = e^{0.06t}$
$\ln(1.66) = 0.06t$
$t = \frac{\ln(1.66)}{0.06} = 8.44$ years

Exercise 1.5 MULTIPLE CHOICE QUESTIONS

1. Which of the following is an example of exponential growth function?

(A) $f(x)=3x+4$ (B) $f(x)=3^x\times 5$ (C) $f(x)=x^3$ (D) $f(x)=x^2$

2. The exponential decay function is expressed by:

(A) $f(x)=a.b^x;0<b<1$ (B) $f(x)=a.b^x;b>1$

(C) $f(x)=a.b^x;0<a<1$ (D) $f(x)=a.b^x;a>1$

3. A population of 8000 increases continuously at a rate of 4% per year. Find the population after 5 years.

$\left(Use\ P=P_0e^{kt}\right)$

(A) 9739 (B) 9730 (C) 9720 (D) 9749

4. The intensity of a sound is $10^{-9}\,W/m^2$. Find its intensity level in decibels.

(Use $\beta=10\log_{10}\left(\frac{I}{I_0}\right), I_0=10^{-12}\,W/m^2$)

(A) 20 Db (B) 30 Db (C) 40 Db (D) 50 Db

5. Rs. 15,000 is invested at 6% per annum compounded annually. Find the amount after 3 years.

(Use $A=P\left(1+\frac{r}{100}\right)^n$)

(A) rs. 17865 (B) rs. 17869 (C) rs. 17925 (D) rs. 15925

6. Rs. 20,000 is invested at 5% per annum compounded continuously. Find the amount after 4 years.

(Use $A=Pe^{rt}$)

(A) Rs. 24293 (B) Rs. 24365 (C) Rs. 24428 (D) Rs. 24596

7. A certain amount of a radioactive substance has half-life of 10 years. If the initial amount is 320 g, find th amount remaining after 30 years.

(Use $A=A_0\left(\frac{1}{2}\right)^{\frac{1}{T}}$)

(A) 20 g (B) 40 g (C) 80 g (D) 160 g

8. The intensity level of a sound is 70 dB. Find its intensity in W/m².

(Use $\beta=10\log_{10}\left(\frac{I}{I_0}\right), I_0=10^{-12}\,W/m^2$)

(A) 10^{-7} (B) 10^{-6} (C) 10^{-5} (D) 10^{-4}

9. Rs. 25,000 is invested at 10% per annum compounded annually. Find the amount after 2 years.

(Use $A=P\left(1+\frac{r}{100}\right)^n$)

(A) Rs. 30250 (B) Rs. 30500 (C) Rs. 30525 (D) Rs. 30625

10. At what rate percent per annum compounded continuously will Rs. 10,000 become Rs. 12,214 in 2 years?

(Use $A=Pe^{rt}$)

(A) 8% (B) 10% (C) 11% (D) 12%

Answers:

	B		A		A	4	B	5	B	6	C	7	A	8	B	9	C	10	